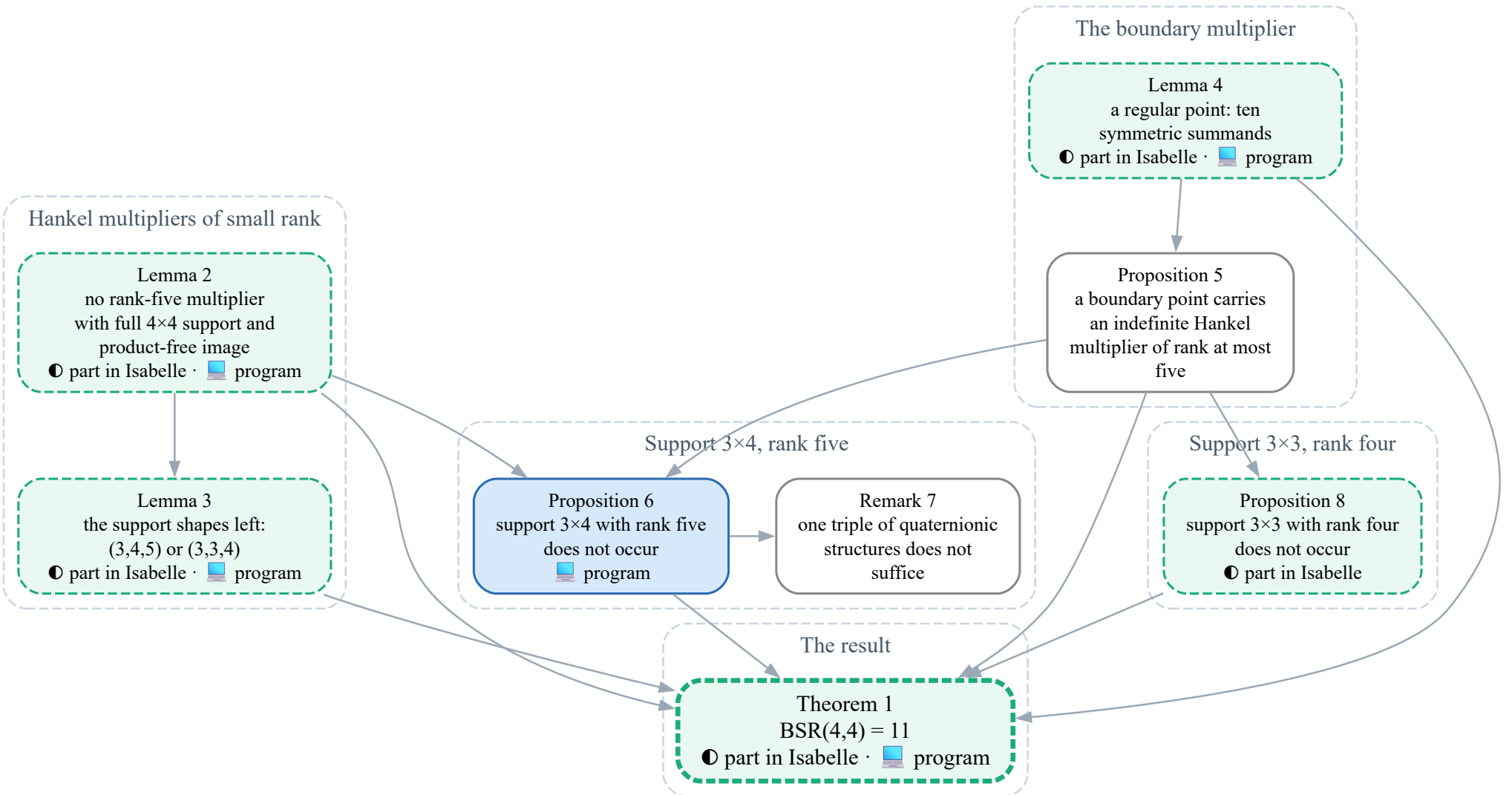
Map of *Every 4×4 biquadratic sum of squares is a sum of eleven squares*

Real algebraic geometry · sums of squares on projective varieties

8 results | **0** entirely in Lean | **5** partly in Lean | **5** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



The exact value **BSR(4,4) = 11**: every biquadratic sum of squares in $x_{1..4}, y_{1..4}$ is a sum of eleven squares of bilinear forms. Qi and Cui gave a form that needs eleven; the previous upper bound was twelve. The proof classifies the Hankel multipliers of rank at most five whose image contains no product (Lemmas 2 and 3) and rules out the two shapes left with second variations built from the alternating forms (Propositions 6 and 8).

🚀 **Start here: Theorem 1**: BSR(4,4) = 11.

Main results:

- **Theorem 1**: BSR(4,4) = 11

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Boundary strategy for Pythagoras numbers (Blekherman–Dunbar–Sinn)** → a boundary point of the sums of eleven squares inside the cone carries an indefinite Hankel multiplier of rank at most five with first and second order conditions — Proposition 5, Lemma 4
- **Linear algebra over C** → commuting symmetric matrices have a common eigenvector; maximal rank in a pencil; isotropic subspaces; Gauss lemma for a rank-one quadratic form — Lemma 2, Lemma 3
- **Alternating forms** → the six alternating forms of $\Lambda^2 R^4$ as second variations: rank $\Psi = 17$ against $4p \leq 16$ — Proposition 6, Proposition 8

What the results serve

- **SOS rank of biquadratic forms** ← the exact value BSR(4,4) = 11, the first case of width four — Theorem 1