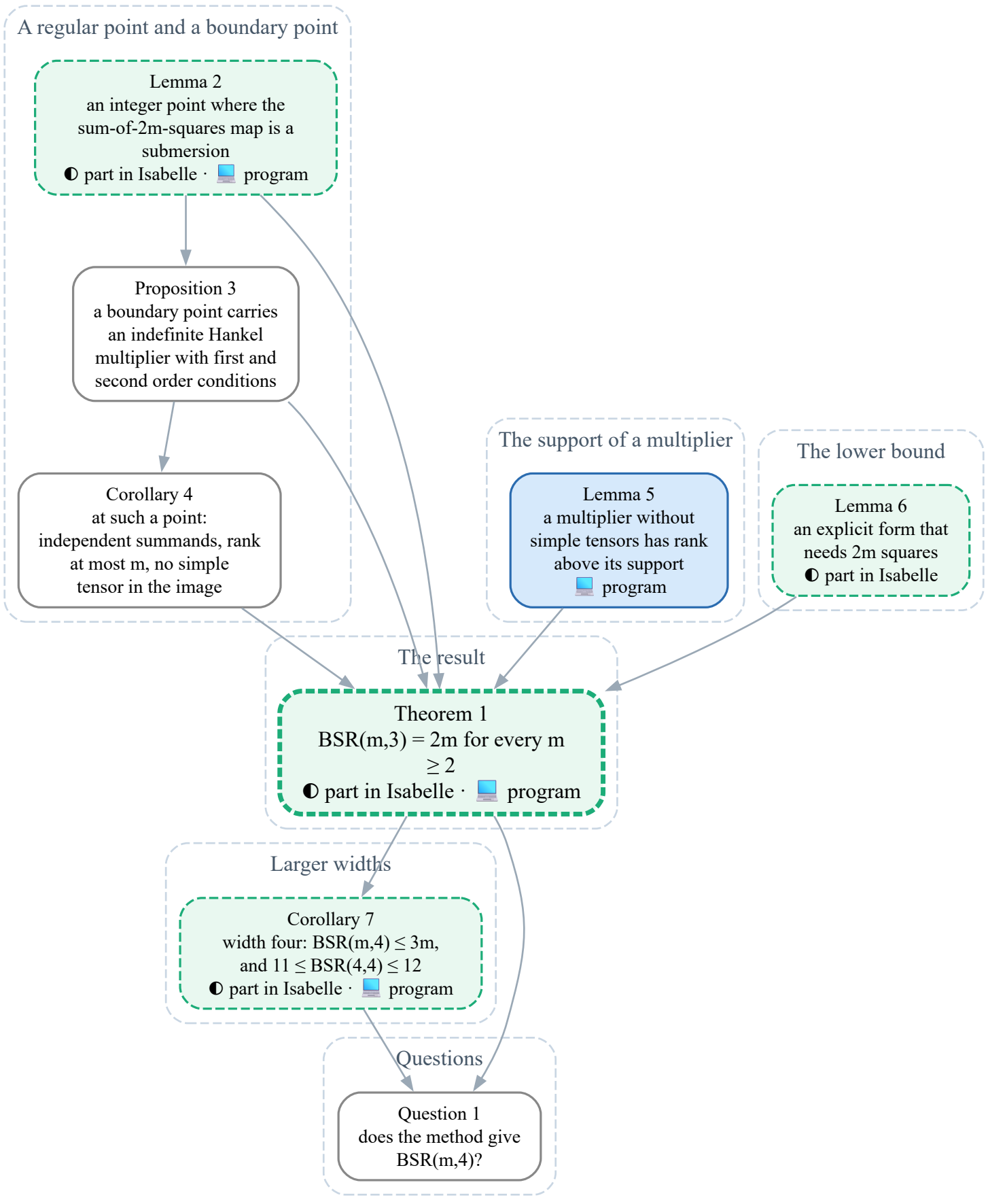
Map of *For m ≥ 2, every m×3 biquadratic sum of squares is a sum of 2m squares*

Real algebraic geometry · sums of squares on projective varieties

8 results | ✅ **0** entirely in Lean | ◐ **4** partly in Lean | 💻 **4** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

The exact sum-of-squares rank of biquadratic forms of width three: every $m\times3$ biquadratic sum of squares is a sum of $2m$ squares of bilinear forms, and some need $2m$. This answers the question of Qi, Löfberg and Chen on the exact values of $\mathrm{BSR}(m,3)$; the cases $m=2,3$ were known, $m\ge4$ is new.

- $\mathrm{BSR}(m,3)=2m$ for every $m\ge2$ (Theorem 1); in particular $\mathrm{BSR}(4,3)=8$.
- An integer point where the map of $2m$ squares is a submersion, in every size (Lemma 2).
- A rank inequality for Hankel multipliers without simple tensors: rank at least the support plus one (Lemma 5), replacing the Gorenstein classification of the boundary strategy of Blekherman, Dunbar and Sinn.
- An explicit form needing $2m$ squares (Lemma 6).

🚀 **Start here: Theorem 1**: BSR(m,3) = 2m for every m ≥ 2.

Main results:

- **Theorem 1**: BSR(m,3) = 2m for every m ≥ 2

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Boundary strategy for Pythagoras numbers (Blekherman–Dunbar–Sinn)** → a generic boundary point of the sums of 2m squares inside the cone carries a Hankel multiplier with first and second order conditions — Proposition 3, Corollary 4
- **Linear algebra over C** → commuting self-adjoint operators have a common eigenvector; a square pencil is singular somewhere on $P^1(C)$; maximal rank in a linear space of matrices — Lemma 5
- **Exact computation** → an integer point where the differential is onto, checked modulo a prime — Lemma 2

What the results serve

- **SOS rank of biquadratic forms** ← the exact value BSR(m,3) = 2m asked by Qi, Löfberg and Chen; BSR(4,3) = 8 — Theorem 1