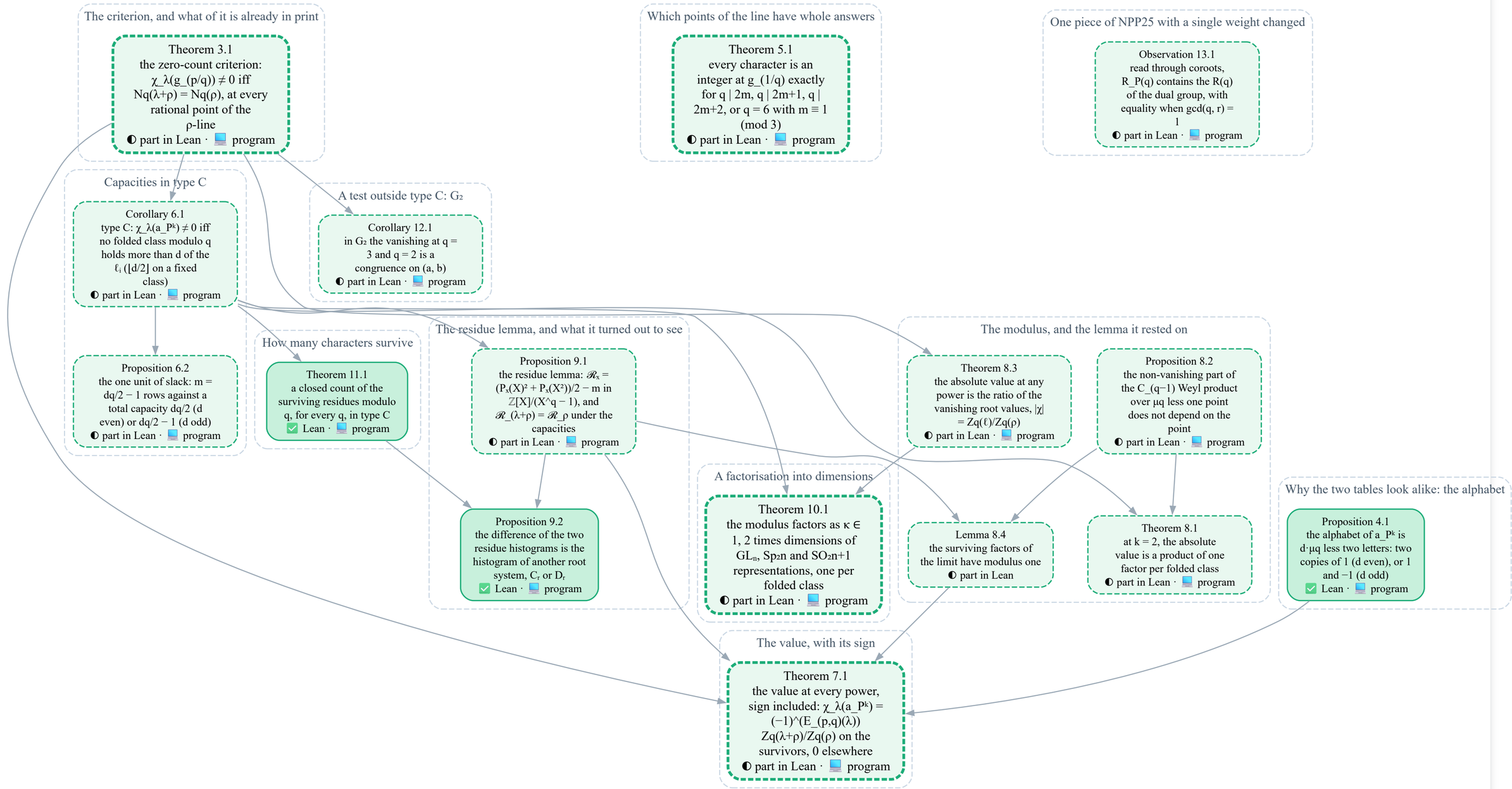
Map of *Integer character values on the ρ-line of Sp(2m)*

Representation theory · characters of compact Lie groups at elements of finite order

16 results | **3** entirely in Lean | **13** partly in Lean | **15** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

Kostant's element a_P of Sp(2m) is one point of the line $\theta \mapsto g_\theta = \exp(2\pi i \theta \rho^\sharp)$, and at it every character is 0 or ±1. The paper studies the rest of the line, and above all the powers a_P^k, where ρ lies on root walls and the alcove argument no longer applies.

- **A classification** (Theorem 5.1): every character of Sp(2m) is an integer at $g_{1/q}$ exactly for q | 2m, q | 2m+1, q | 2m+2, and q = 6 with m ≡ 1 (mod 3); the powers of a_P are the family q | 2m+2.
- **A vanishing criterion** at every rational point (Theorem 3.1): $\chi_\lambda(g_{p/q}) \neq 0$ iff $N_q(\lambda+\rho) = N_q(\rho)$; in type C, capacities of folded residue classes, with one unit of slack when d is even (Corollary 6.1, Proposition 6.2).
- **The value at every power, sign included** (Theorem 7.1): $(-1)^{E_{p,q}(\lambda)}\, Z_q(\lambda+\rho)/Z_q(\rho)$, through a residue lemma that needs no pairing of roots (Proposition 9.1); its modulus is $\kappa \in \{1, 2\}$ times dimensions of GL_n, Sp_{2n} and SO_{2n+1}, one per folded class (Theorem 10.1); a closed count of the survivors for every q (Theorem 11.1); G_2 at q = 3 (Corollary 12.1).

What the paper says is not new. At a_P itself the criterion and the sign are Corollary 3.2 of Cellini–Möseneder Frajria–Papi; read on coroots the count is Corollary 3.7 of Nadimpalli–Pattanayak–Prasad, the same count root by root when gcd(q, r) = 1; the principal specialisation is classical; at q = 2 the factorisation into dimensions has the two-block form of Theorem 5.1 of Karmakar, and in G_2 the case q = 2 is his Proposition 2; the necessity in Theorem 5.1 also follows from Gannon–Schopieray, Lemma 4.6.

What is in Lean, and what is not (the paper's section «Formalization», version 2). In Lean 4 with Mathlib (commit 6283de8): the type-C_m statements on the root products F_x and their quotient Q, including that F_ρ divides F_ℓ, the spectral classification behind Theorem 5.1, and the rest of that list; each statement below shows its declarations and their scope. Outside Lean: the identification of Q(ζ) with the character (Weyl's formula), the equivalence between spectral and character integrality, that the $\nu^{(c)}$ are partitions, and the step from a dominant λ to ℓ.

🚀 **Start here: Theorem 3.1**: the zero-count criterion: χ_λ(g_(p/q)) ≠ 0 iff Nq(λ+ρ) = Nq(ρ), at every rational point of the ρ-line.

Main results:

- **Theorem 3.1**: the zero-count criterion: χ_λ(g_(p/q)) ≠ 0 iff Nq(λ+ρ) = Nq(ρ), at every rational point of the ρ-line
- **Theorem 5.1**: every character is an integer at g_(1/q) exactly for q | 2m, q | 2m+1, q | 2m+2, or q = 6 with m ≡ 1 (mod 3)
- **Theorem 7.1**: the value at every power, sign included: χ_λ(a_P^k) = (−1)^(E_(p,q)(λ)) Zq(λ+ρ)/Zq(ρ) on the survivors, 0 elsewhere

- **Theorem 10.1**: the modulus factors as $\kappa \in 1, 2$ times dimensions of GL_n, Sp_{2n} and SO_{2n+1} representations, one per folded class

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Weyl character formula and root systems** → the principal specialisation read on the line of ρ (root statistic), and the order of vanishing of each factor at a root of unity — Theorem 3.1, Theorem 8.3, Theorem 7.1, 13.1
- **Galois theory of cyclotomic fields** → integral characters force a Galois-stable spectrum; a strict inequality for Dirichlet kernels classifies the stable segments — Theorem 5.1
- **Combinatorics of residues** → folded residue classes modulo q, capacities by marginal cost, and the closed count of the survivors — Corollary 6.1, Proposition 6.2, Theorem 11.1
- **Products of roots of unity** → Vandermonde and deletion arguments; the residue identity $\mathscr{R}_x = (P_{x^2} + P_x(X^2))/2 - m$, the adjoint character minus the rank — Proposition 8.2, Lemma 8.4, Proposition 9.1, Proposition 9.2
- **Representations of classical groups** → Weyl dimension formulas for GL_n, Sp_2n and SO_2n+1, one block per folded class — Theorem 10.1
- **Formal verification** → Lean 4 + Mathlib: the residue identity, the capacity count and the slack over ZMod q; the stable segments (Amiot) — Proposition 9.1, Corollary 6.1, Proposition 6.2, Theorem 5.1

What the results serve

- **Characters at elements of finite order** ← the line of Kostant, Cellini–Möseneder Frajria–Papi and Nadimpalli–Pattanayak–Prasad, carried to the powers of a_P, where ρ is no longer regular — Theorem 3.1, Theorem 7.1, Theorem 8.3
- **Classical characters twisted by roots of unity** ← the alphabet of Ayyer–Kumari, Albion and Lecouvey short of two letters, and the constant κ it puts in front of the dimensions — Proposition 4.1, Theorem 10.1