

Map of *Characters of $Sp(2m)$ at the powers of Kostant's element*



Representation theory · characters of Lie groups

05E10

20G05

17B22

05A15

12 results

✓ 0 formalized in Lean

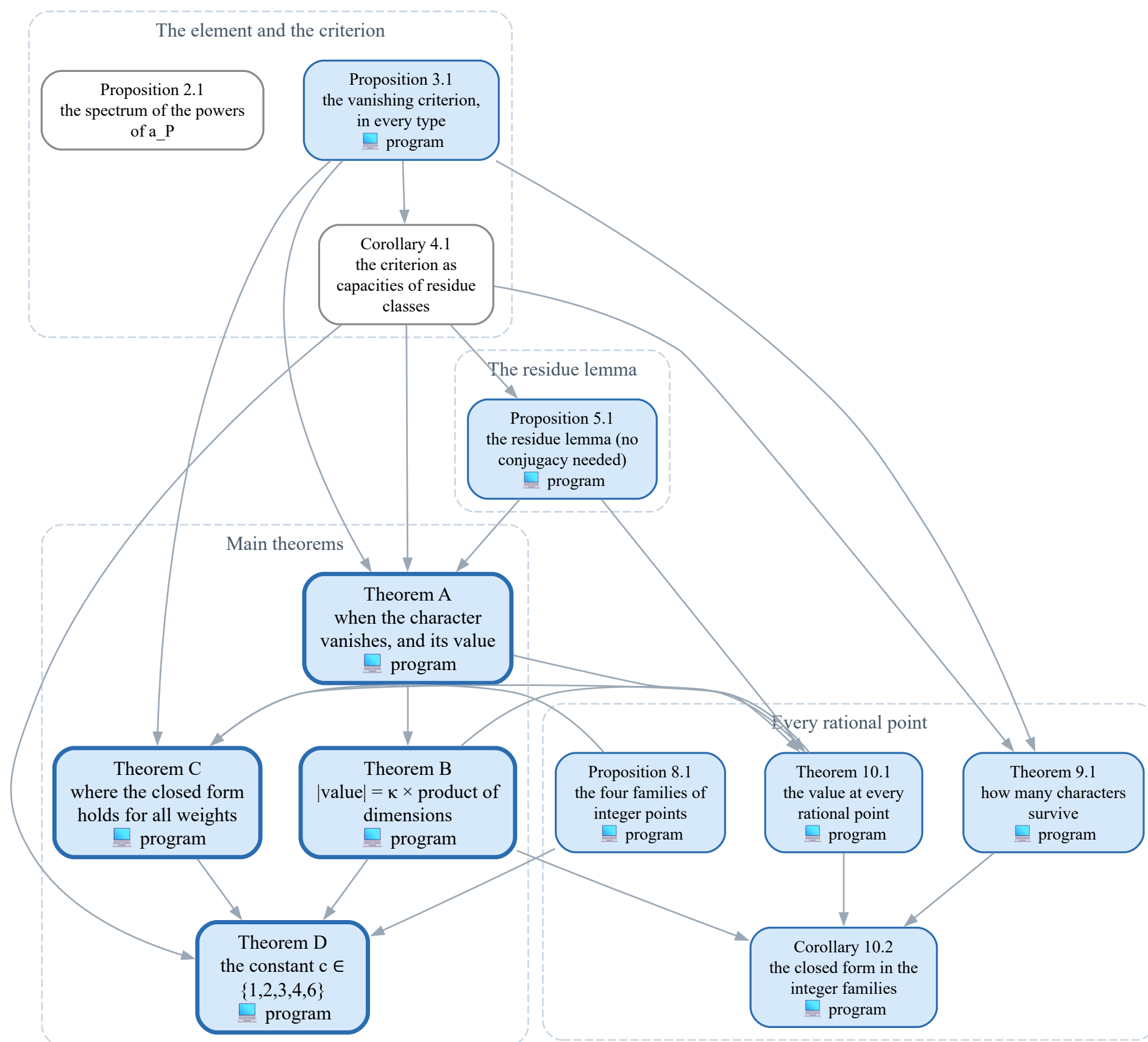
0 with a part in Lean

10 checked by a program

written proof only

(a result can carry several tags)

Dependencies in the written proof: an arrow $A \rightarrow B$ means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's “Evidence & scope” block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

Kostant's element a_p is a point where every irreducible character of a compact simple group takes a value in $\{0, \pm 1\}$. The paper determines the irreducible characters of $Sp(2m)$ at **every power** a_p^k .

- **When a character vanishes, and its value** (Theorem A): it vanishes exactly when $\lambda + \rho$ lies on more walls of the modular root arrangement than ρ does, equivalently when some folded residue class holds more entries than its capacity; otherwise the value is an explicit signed ratio of root products.
- **The absolute value is a product of dimensions** of representations of GL, Sp and odd SO, times $\kappa \in \{1, 2\}$ (Theorem B).
- **Every rational point of the same line:** a non-vanishing character is, up to sign and that factor, such a product times a character of $Sp(2s)$ or $SO(2s)$, $s \leq m$, at a regular element (Theorem 10.1); the points where the closed form holds for all weights are listed (Theorem C), and at the points where all characters are integers the remaining factor is 1, 2, 3, 4 or 6 (Theorem D).

Known and new, as the paper states it. For k prime to $2m+2$, Theorem A is due to Cellini, Möseneder Frajria and Papi; for $\gcd(k, 2m+2)$ odd, Theorems A and B follow from Albion's factorisations of universal characters, with the sign added. New: for $\gcd(k, 2m+2)$ even and $q \geq 3$, the values on the classes not conjugate to that of ρ and the factor κ ; the list of points in Theorem C and the exclusion of the others; in Theorem D, the constants for $q \geq 4$ even with $q \mid 2m$, $q \nmid m$, and for $q = 6$ with $m \equiv 4 \pmod{6}$.

Key tool: an identity of residue multisets (Proposition 5.1), which takes the place of the conjugacy hypothesis in the work of Nadimpalli, Pattanayak and Prasad.

 **Start here:** [Theorem A](#): when the character vanishes, and its value.

Main results:

- [Theorem A](#): when the character vanishes, and its value
- [Theorem B](#): $|\text{value}| = \kappa \times \text{product of dimensions}$
- [Theorem C](#): where the closed form holds for all weights
- [Theorem D](#): the constant $c \in \{1, 2, 3, 4, 6\}$

Fields: where the paper lives and what it connects

Tools taken from other branches

- **Root systems** $\rightarrow$ walls of the modular root arrangement: the count criterion, proved by the argument of Nadimpalli–Pattanayak–Prasad (the alcove argument of Cellini–Möseneder Frajria–Papi needs ρ regular and no longer applies) — [Proposition 3.1](#), [Theorem A](#)
- **Elementary number theory** $\rightarrow$ residues modulo q in folded classes; an identity of residue multisets — [Corollary 4.1](#), [Proposition 5.1](#)
- **Algebraic combinatorics** $\rightarrow$ partitions, Weyl dimension formulas; the factorisations of universal and classical characters (Albion, Ayyer–Kumari) — [Theorem B](#)
- **Algebraic number theory** $\rightarrow$ Galois action on the spectrum: which points give integer characters (a lemma of Gannon–Schopieray) — [Proposition 8.1](#)
- **Enumeration** $\rightarrow$ counting the non-vanishing residues — [Theorem 9.1](#)

What the results serve

- **Characters at elements of finite order** $\leftarrow$ Kostant's $\{0, \pm 1\}$ theorem, extended to every power of a_p ; the question of Nadimpalli, Pattanayak and Prasad on vanishing and values at torsion elements, addressed for these elements of $\mathrm{Sp}(2m)$ — [Theorem A](#), [Theorem B](#)
- **Reeder's constants** $\leftarrow$ their values on the classes not conjugate to that of ρ — [Theorem A](#), [Proposition 5.1](#)
- **Factorisations of classical characters** $\leftarrow$ a factor κ and a residual character of type C or D at every rational point of the line — [Theorem B](#), [Theorem 10.1](#), [Theorem D](#)