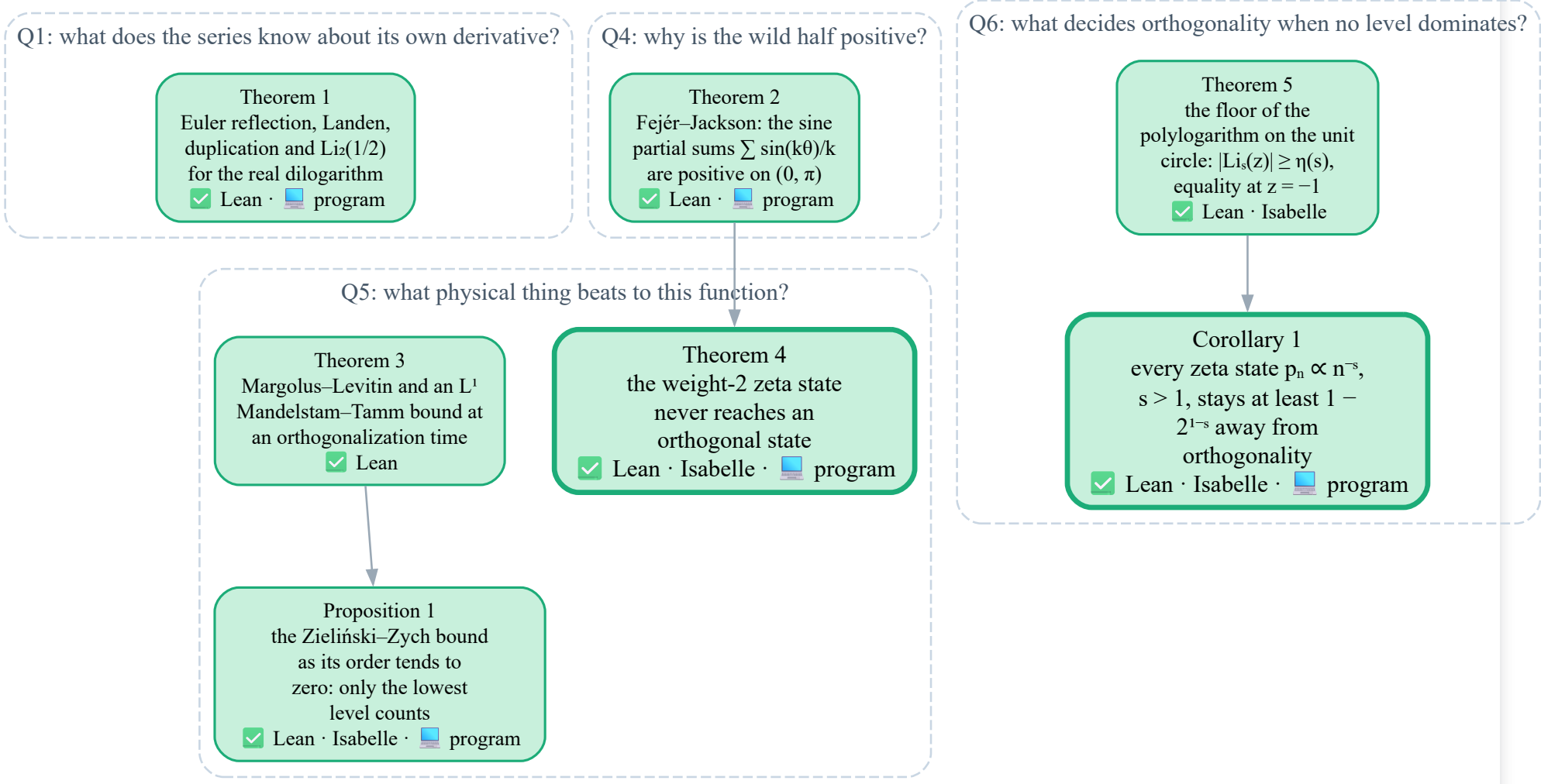
Map of *The Clock That Never Ticks: a machine-checked path from the dilogarithm to quantum speed limits in Lean 4*

Special functions · the dilogarithm and the Clausen function

7 results | ✅ 7 entirely in Lean | ◐ 0 partly in Lean | ✅ 4 in Isabelle | 💻 5 checked by a program | written proof only

(a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

A sorry-free Lean 4 path from the dilogarithm, which Mathlib cites but does not define, to a quantum state that never orthogonalizes; version 2 adds its exact floor, in Lean and in Isabelle.

- **The real dilogarithm** with Euler's reflection, Landen's transformation, the duplication formula and $\mathrm{Li}_2(1/2)$ (Theorem 1); the golden-ratio values and the Lee–Yang $c_{\mathrm{eff}} = 2/5$ from a 3×3 linear system, without the five-term relation.
- **The circle**: the Clausen function and Catalan's constant $G = \mathrm{Cl}_2(\pi/2)$; the Fejér–Jackson inequality (Theorem 2) and, by Abel summation without asymptotics, $\mathrm{Cl}_2(\theta) \geq \sin(\theta)/2$ on $(0, \pi)$.
- **Speed limits**: Margolus–Levitin and an L^1 form of Mandelstam–Tamm for a scalar spectral measure (Theorem 3).
- **The assembly** (Theorem 4): populations $6/(\pi^2 n^2)$ on levels $E_n = n$ have divergent first and second energy moments, so both bounds give nothing, and yet the state never reaches an orthogonal state. Version 2 gives the one-line reason (the lowest level carries more than half of the weight) and shows that the Zieliński–Zych limits see exactly that (Proposition 1).
- **The exact floor** (Theorem 5, Corollary 1): $|S| \geq 1 - 2^{1-s}$ for every zeta state $p_n \propto n^{-s}$, also where no level dominates.
- The mathematics is classical; the paper claims the formalization, the assembly and the place of the zeta state among the speed limits.

🚀 **Start here: Theorem 1**: Euler reflection, Landen, duplication and $\mathrm{Li}_2(1/2)$ for the real dilogarithm.

Main results:

- **Theorem 4**: the weight-2 zeta state never reaches an orthogonal state
- **Corollary 1**: every zeta state $p_n \propto n^{-s}$, $s > 1$, stays at least $1 - 2^{1-s}$ away from orthogonality

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Power series and real analysis** → term-by-term differentiation, the Weierstrass M-test, mean-value constancy, the boundary limit $\ln x \cdot \ln(1-x) \to 0$ — Theorem 1
- **Fourier series and positive trigonometric sums** → Mathlib's Bernoulli–Fourier expansion for the cosine half; a division-free Dirichlet telescope and an interior-minimum induction; Abel summation without asymptotics — Theorem 2, Theorem 4

- **Quantum mechanics** → speed limits stated for a scalar autocorrelation of a discrete spectral measure, with the cosine inequality $\cos x \geq 1 - (2/\pi)(x + \sin x)$ and its fractional-moment version (Zieliński–Zych) — Theorem 3, Proposition 1
- **Measure and integration (Hausdorff moment sequences)** → the coefficients of Li_s as moments of a positive measure on $[0, 1)$ (a Stieltjes transform), integrated against $\mathrm{Re}\, 1/(1 - tz) \geq 1/(1 + t)$, and Abel's theorem to reach the circle — Theorem 5

What the results serve

- **Quantum speed limits** ← a state with divergent first and second energy moments that never orthogonalizes; what the moment-based limits see (only the lowest level) and the exact floor they miss — Theorem 4, Proposition 1, Corollary 1
- **Dilogarithm identities** ← the golden-ratio values and the Lee–Yang $c_{\mathrm{eff}} = 2/5$ without the five-term relation; the five-term relation is the companion paper — Theorem 1
- **Power series with positive coefficients on the unit circle** ← the floor of $|\mathrm{Li}_s|$ at $z = -1$ for every $s > 0$ (classical: Fejér 1937); which shape conditions put the smallest modulus at $z = -1$, with exact counterexamples — Theorem 5, Corollary 1