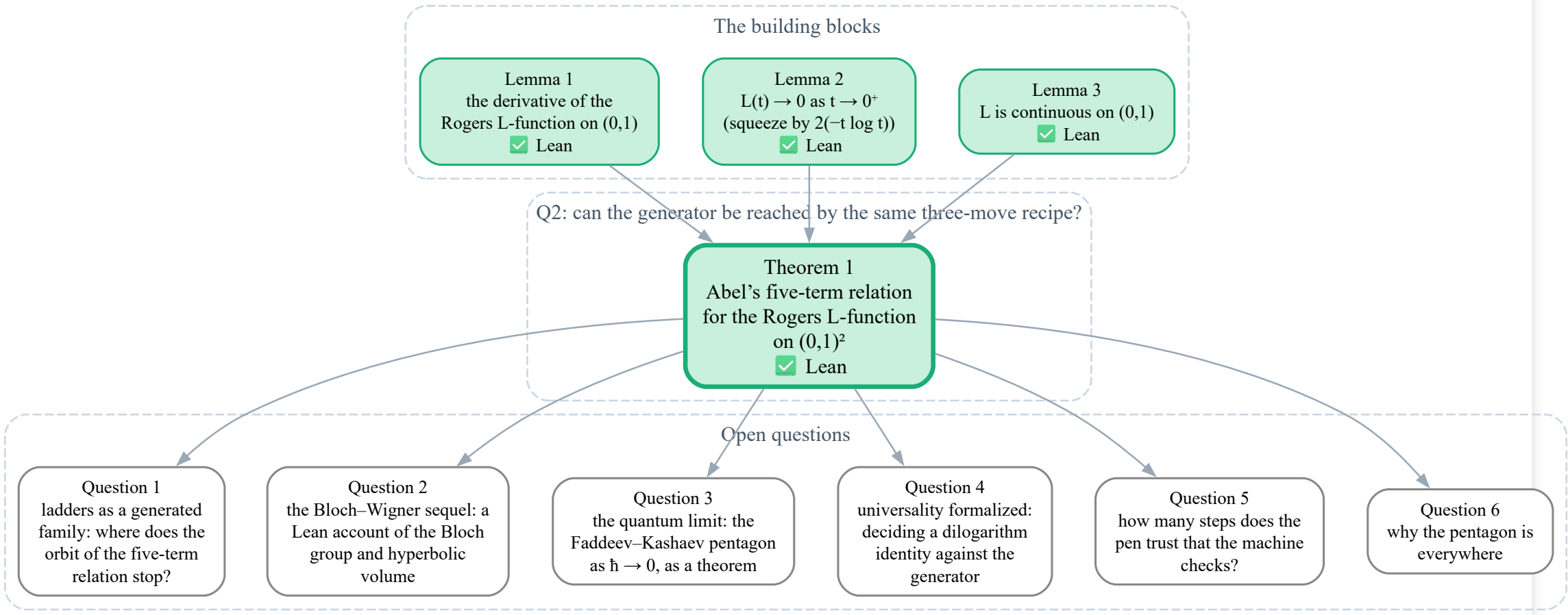
Map of *The Pentagon, Machine-Checked: Abel's five-term relation for the dilogarithm in Lean 4*

Special functions · the dilogarithm

10 results | ✅ **4** entirely in Lean | ◐ **0** partly in Lean | 💻 **0** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

A machine-checked proof of the identity that generates the weight-2 theory of the dilogarithm, on the dilogarithm built in Lean by the companion paper.

- **Abel's five-term relation** (Theorem 1), sorry-free in Lean 4: for x, y in (0,1), $L(x) + L(y) = L(xy) + L(x(1-y)/(1-xy)) + L(y(1-x)/(1-xy))$, with L the Rogers L-function.
- **The classical route, made formal**: the five-term difference has zero derivative in x (nine logarithms collapse into a basis of five), so it is constant; its value is read off as $x \to 0^+$, where $\log t \cdot \log(1-t) \to 0$ by a squeeze.
- **Three reusable blocks**: the derivative of L on (0,1), $L(t) \to 0$ as $t \to 0^+$, continuity of L on (0,1).
- The mathematics is classical (Spence, Abel, Rogers); the paper claims the formalization, after a documented search of proof-assistant libraries.

🚀 **Start here: Theorem 1**: Abel's five-term relation for the Rogers L-function on $(0,1)^2$.

Main results:

- **Theorem 1**: Abel's five-term relation for the Rogers L-function on $(0,1)^2$

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Real analysis** → vanishing derivative on (0,1) gives a constant (mean-value theorem); the boundary limit $\log t \cdot \log(1-t) \to 0$ by the squeeze $2(-t \log t)$ — Theorem 1
- **Formal verification** → Lean 4 + Mathlib: the nine-into-five logarithm cancellation, closed by field_simp and ring — Theorem 1

What the results serve

- **Functional equations of the dilogarithm** ← by Wojtkowiak and de Jeu, every rational functional equation of Li_2 follows from this one (cited, not formalized) — Theorem 1
- **Bloch group, cluster algebras, quantum dilogarithm** ← the defining relation of the Bloch group and the pentagon of cluster mutations (context; not formalized here)