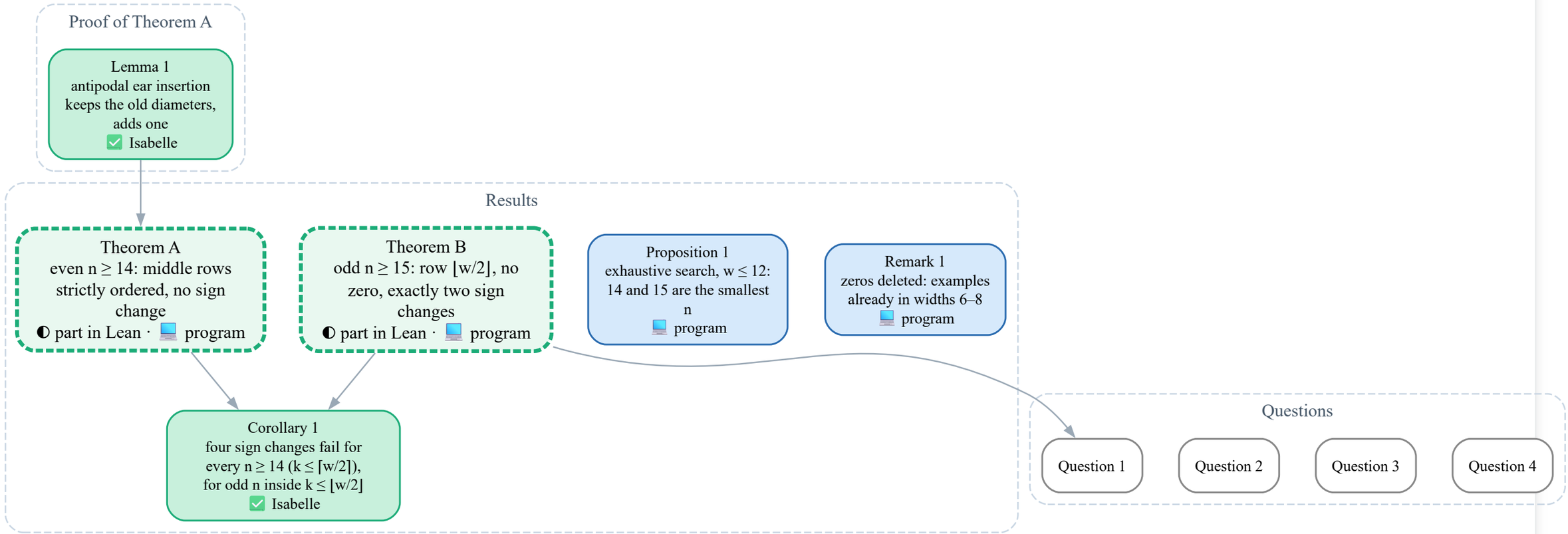
Map of *Integer frieze patterns and Tabachnikov's four sign changes*

Combinatorics · frieze patterns and triangulations of polygons

10 results | ✅ **0** entirely in Lean | ◐ **2** partly in Lean | ✅ **2** in Isabelle | 💻 **4** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

A negative answer to Problem 1 of Tabachnikov (*A four-vertex theorem for frieze patterns?*, 2018) for frieze patterns of positive integers, which the question left open:

- for every even $n \geq 14$, two integer frieze patterns whose middle rows are strictly ordered (no sign change);
- for every odd $n \geq 15$, two whose rows $\lfloor w/2 \rfloor$ differ with no zero and exactly two sign changes, inside the range $1 \le k \le [w/2]$ of the question;
- an exhaustive search showing that 14 and 15 are the smallest values of n for the two phenomena, when zeros may count as sign changes.

Both families are proved for every n and checked in Isabelle/HOL; the two smallest pairs are also checked in Lean.

🚀 **Start here: Lemma 1**: antipodal ear insertion keeps the old diameters, adds one.

Main results:

- **Theorem A**: even n ≥ 14: middle rows strictly ordered, no sign change
- **Theorem B**: odd n ≥ 15: row ⌊w/2⌋, no zero, exactly two sign changes

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Linear algebra in the plane** → frieze patterns as unimodular lattice polygons: every entry a 2×2 determinant, an ear a sum of two vectors — Lemma 1, Theorem A, Theorem B
- **Exhaustive search** → all pairs of integer frieze patterns of width ≤ 12, on a GPU, rechecked by three independent constructions — Proposition 1

What the results serve

- **Four-vertex theorems for discrete curves** ← the integer case of Tabachnikov's question, answered negatively from n = 14 on — Corollary 1