What a Determinant's Derivative Knows: Jacobi's Formula and Newton's Identity for Matrix Traces in Lean 4

Linear algebra · characteristic polynomials (formalized)

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Statements

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At a glance

What the paper contributes

A machine-checked formalization, in Lean 4 / Mathlib, of Jacobi's formula and Newton's identity for matrix traces, composed with Bass's formula. The mathematics is classical; the contribution is the formalization.

- **Jacobi's formula** in its algebraic form, $(\det M)' = \mathrm{tr}(\mathrm{adj}\, M \cdot M')$ for a matrix of polynomials over a commutative ring (Theorem 1), for which the paper found no prior proof in any proof assistant. Axioms: `propext`, `Classical.choice`, `Quot.sound`.
- **Newton's identity for matrix traces** to all orders, eigenvalue-free: $(\mathrm{charpolyRev}\, M)' = -\mathrm{charpolyRev}\, M \cdot \Sigma_k \mathrm{tr}(M^{k+1})\, X^k$ (Theorem 2), through the resolvent series $(I - XM)^{-1} = \Sigma_k M^k X^k$ over formal power series (Lemma 1).
- **Matrix powers count walks**, also for directed graphs (Lemma 2), with the trace rewritten over cyclic walks and its weight invariant under rotation.
- **Composed with Bass's formula**: the generating function of the non-backtracking counts $N_k = \mathrm{tr}(B^k)$ from the adjacency data (A, D) (Theorem 3), and $\mathrm{tr}(B^k)$ counts rooted closed non-backtracking walks (Theorem 4).
- **The boundary**, as the paper states it: rooted walks, not geodesic classes; the prime-geodesic Euler product is mapped, not built.

🚀 **Start here: Theorem 1**: Jacobi's formula: $(\det M)' = \mathrm{tr}(\mathrm{adj}\, M \cdot M')$ for a matrix of polynomials.

Main results:

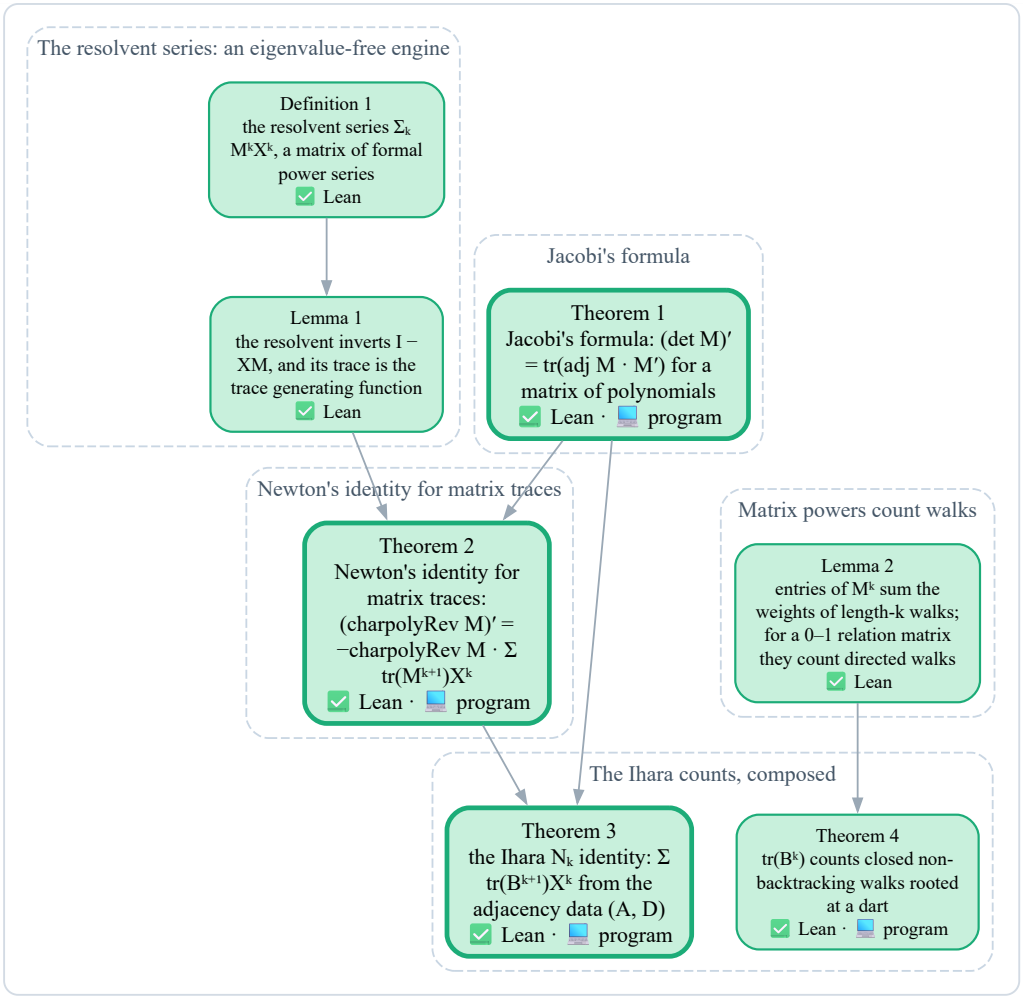
- **Theorem 1**: Jacobi's formula: $(\det M)' = \mathrm{tr}(\mathrm{adj}\, M \cdot M')$ for a matrix of polynomials
- **Theorem 2**: Newton's identity for matrix traces: $(\mathrm{charpolyRev}\, M)' = -\mathrm{charpolyRev}\, M \cdot \Sigma\, \mathrm{tr}(M^{k+1})X^k$
- **Theorem 3**: the Ihara N_k identity: $\Sigma\, \mathrm{tr}(B^{k+1})X^k$ from the adjacency data (A, D)

7 results | ✅ 7 entirely in Lean | ◐ 0 partly in Lean

💻 4 checked by a program | written proof only | (a result can carry several tags)

🗺️ Map of the results (click a box to go to it)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



Setting (copied from the paper, §2)

For a matrix M whose entries are polynomials in a formal variable, write M' for the entrywise derivative and $\det M$ for the determinant (a polynomial). $\mathrm{adj}\, M$ is the adjugate (the transpose of the cofactor matrix) *(§1)*. *(§4)* Write $\mathrm{charpolyRev}\, M$ for the reversed characteristic polynomial $\det(I - XM)$—a polynomial with constant term 1, hence a unit in the power-series ring. *(§6)* Let B be Hashimoto's operator, A the adjacency and D the degree matrix; Bass's formula reads $(1-X^2)^{|V|}\det(I-XB) = (1-X^2)^{|E|}\det(I-XA+X^2(D-I))$.

What this blueprint shows.

The numbered statements of the paper, with their proofs, dependencies and what checked them, under the sections that hold them (with the paper's numbers), and the figures and tables they use. The full text is in the PDF of the paper.

2 Jacobi's formula

Theorem 1. Lean **(Jacobi's formula, in Lean 4)**✓ p. 3

> *For an $n \times n$ matrix M of polynomials over a commutative ring,*

$$(\det M)' \;=\; \operatorname{tr}\!\big(\operatorname{adj}M \cdot M'\big).$$

The statement is sorry-free (`Matrix.derivative_det`); `#print axioms` reports only `propext`, `Classical.choice`, `Quot.sound`.

Lean · ours show

- `Matrix.derivative_det`

✅ Checked when this page was built: Jacobi.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — derivative_det: propext, Classical.choice, Quot.sound.
Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean Jacobi.lean` (SHA-256 of the compiled file: `6ec866e48406ee3e`…; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 1 result, standard axioms only

Proof: in this note · **Formalized:** Lean 4, by the author (public formalizations repository) · **Covers:** the whole statement · **Cite:** Theorem 1, p. 3, doi:10.5281/zenodo.20578470

🆕 **Classical and new:** classical; the paper claims the first machine-checked proof it is aware of of the algebraic formula (N1), after searching Mathlib, the AFP, Coq/Rocq, HOL Light and Mizar; Mathlib had only the analytic derivative of det.

↳ Used in: Theorem 2 (p. 6), Theorem 3 (p. 8)

Proof

Formalized in Lean (see the linked declarations).

Evidence & scope

- **Program:** Jacobi on one 3×3 matrix of polynomials, exact (SymPy). `verify_jacobi_newton.py`

3 The resolvent series: an eigenvalue-free engine

Definition 1. Lean (Resolvent series)✓ p. 3

For an $n \times n$ matrix M over a commutative ring, the *resolvent series* is the matrix of formal power series $\left(\sum_k M^k X^k\right)$, written `resolventSeries M`.

Lean · ours show

- `Matrix.resolventSeries`

✅ Checked when this page was built: Jacobi.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — resolventSeries: propext, Classical.choice, Quot.sound.

Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean Jacobi.lean` (SHA-256 of the compiled file: `6ec866e48406ee3e…`; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 1 result, standard axioms only

Cite: Definition 1, p. 3, doi:10.5281/zenodo.20578470

↳ Used in: Lemma 1 (p. 5)

Proof

Formalized in Lean (see the linked declarations).

Lemma 1. ✅ Lean

(The resolvent inverts $I - XM$, and its trace is the trace generating function)

✓ 📕 p. 3

$$(I - XM)\cdot\textstyle\sum_k M^kX^k \;=\; I, \qquad \operatorname{tr}\!\left(\textstyle\sum_k M^kX^k\right) \;=\; \textstyle\sum_k \operatorname{tr}(M^k)\,X^k.$$

Both are `sorry`-free (`one_sub_smul_mul_resolventSeries`, `trace_resolventSeries`). The inverse identity is proved from the fixed-point equation $R = I + XM\cdot R$ by a single rearrangement—no eigenvalues, no algebraic closure, over any commutative ring.

Lean · ours show

- `Matrix.one_sub_X_smul_mul_resolventSeries`
- `Matrix.trace_resolventSeries`

✅ Checked when this page was built: Jacobi.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — one_sub_X_smul_mul_resolventSeries, trace_resolventSeries: propext, Classical.choice, Quot.sound.

Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean Jacobi.lean` (SHA-256 of the compiled file: `6ec866e48406ee3e…`; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 2 results, standard axioms only

Proof: in this note · **Formalized:** Lean 4, by the author (public formalizations repository) · **Covers:** the whole statement · **Cite:** Lemma 1, p. 3, doi:10.5281/zenodo.20578470

🆕 **Classical and new:** the paper counts the resolvent series among the pieces not in the library (N3).

↳ Used in: Theorem 2 (p. 6)

Proof

Formalized in Lean (see the linked declarations).

4 Newton's identity for matrix traces

Theorem 2. Lean **(Newton's identity for matrix traces, in Lean 4)**✓

p. 4

For an $n \times n$ matrix M over a commutative ring, in the ring of formal power series,

$$(\mathrm{charpolyRev}\, M)' \;=\; -\,\mathrm{charpolyRev}\, M \cdot \sum_{k\ge 0} \mathrm{tr}(M^{k+1})\, X^k.$$

Equivalently $\sum_{k\ge 1} \mathrm{tr}(M^k) X^k = -X\,(\mathrm{charpolyRev}\, M)'/\,\mathrm{charpolyRev}\, M$*, the logarithmic derivative. The statement is* `sorry`*-free (*`Matrix.charpolyRev_logDeriv`*), over any commutative ring, eigenvalue-free. At the linear coefficient it specializes to the lemma Mathlib already had,* $\mathrm{tr} M = -\,[X^1]\,\mathrm{charpolyRev}\, M$.

Lean · ours **show**

- `Matrix.coe_derivative_charpolyRev_eq_neg_mul_mk_trace_pow`
- `Matrix.derivative_coe_charpolyRev`

✅ Checked when this page was built: Jacobi.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — coe_derivative_charpolyRev_eq_neg_mul_mk_trace_pow, derivative_coe_charpolyRev: propext, Classical.choice, Quot.sound.
Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean Jacobi.lean` (SHA-256 of the compiled file: `6ec866e48406ee3e`…; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 2 results, standard axioms only

Proof: in this note · **Formalized:** Lean 4, by the author (public formalizations repository) · **Covers:** the whole statement · **Cite:** Theorem 2, p. 4, doi:10.5281/zenodo.20578470

🆕 **Classical and new:** classical; the matrix-trace form, which Mathlib had only at the linear coefficient (N2).

↳ Used in: Theorem 3 (p. 8)

Proof

Formalized in Lean (see the linked declarations).

🛡️ **Evidence & scope**

- **Program:** the identity modulo X^8 for one 3×3 and one 2×2 integer matrix, exact (SymPy). `verify_jacobi_newton.py`

5 Matrix powers count walks

Lemma 2. ✅ Lean **(Powers count walks)** ✓ p. 5

> *For a matrix M over a commutative semiring, the (i, j) entry of M^k is the sum, over length-k vertex walks from i to j, of the product of the edge weights (`pow_apply_eq_sum`). For the 0–1 adjacency matrix of a decidable relation—a finite directed graph—this is a genuine count: $((\text{adjacency})^k)_{ij}$ is the number of length-k directed walks from i to j (`relMatrix_pow_apply`). Both are `sorry`-free.*

Lean · ours **show**

- `Matrix.pow_apply_eq_sum`
- `Matrix.relMatrix_pow_apply`
- `Matrix.trace_pow_eq_sum_cyclicProd`
- `Matrix.cyclicProd_comp_finRotate`
- `Matrix.cyclicProd_comp_finRotate_iterate`

✅ Checked when this page was built: WalkCount.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — pow_apply_eq_sum, relMatrix_pow_apply, trace_pow_eq_sum_cyclicProd, cyclicProd_comp_finRotate, cyclicProd_comp_finRotate_iterate: propext, Classical.choice, Quot.sound. Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean WalkCount.lean` (SHA-256 of the compiled file: `7b1bf7dcaf36423e`…; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 5 results, standard axioms only

Proof: in this note · **Formalized:** Lean 4, by the author (public formalizations repository) · **Covers:** the whole statement · **Cite:** Lemma 2, p. 5, doi:10.5281/zenodo.20578470

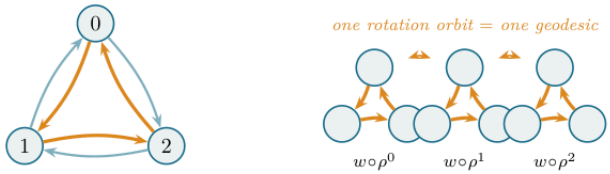
🆕 **Classical and new:** the directed-graph walk count is among the pieces not in the library (N3).

↳ Used in: Theorem 4 (p. 9)

Proof

Formalized in Lean (see the linked declarations).

Figure 2



one closed nb-walk w

Closed non-backtracking walks, and the rotation that groups them. *Left:* the triangle K_3 has six darts; one closed non-backtracking walk of length 3 is shown in amber (the reverse darts, in grey, would backtrack). *Right:* rotating the basepoint by the cyclic shift $\rho =$ `finRotate` permutes the three rooted walks of this orbit while leaving the cyclic weight fixed (`cyclicProd_comp_finRotate`)—so a single *geodesic* accounts for an orbit of rooted walks. Here $\mathrm{tr}(B^3) = 6$: two orbits (the cycle and its reverse), three basepoints each.

6 The Ihara counts, composed

Theorem 3. Lean **(The Ihara N_k identity, in Lean 4)**✓ p. 5

With $Q = \det(I - XA + X^2(D - I))$, in the power-series ring,

$$\sum_k \mathrm{tr}(B^{k+1})X^k \;=\; \frac{\det(I - XB)\left((1-X^2)^{|V|}\right)' - (1-X^2)^{|E|}Q' - Q\left((1-X^2)^{|E|}\right)'}{(1-X^2)^{|V|}\det(I - XB)}.$$

The two $(1 - X^2)$-powers and both determinants are units, so the division is the formal `invOfUnit`*; the identity is* `sorry`*-free (*`ihara_Nk_explicit`*). It expresses the non-backtracking counts $N_k = \mathrm{tr}(B^k)$ purely through the adjacency data (A, D).*

Lean · ours show

- `SimpleGraph.ihara_Nk_explicit`

✅ Checked when this page was built: TraceFormula.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — ihara_Nk_explicit: propext, Classical.choice, Quot.sound. Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean TraceFormula.lean` (SHA-256 of the compiled file: `2a7348dcd68d3e94`…; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 1 result, standard axioms only

Proof: in this note · **Formalized:** Lean 4, by the author (public formalizations repository) · **Covers:** the whole statement · **Cite:** Theorem 3, p. 5, doi:10.5281/zenodo.20578470

🆕 **Classical and new:** composition of Jacobi and Newton with the author's Bass formalization; Bass's theorem is classical.

↳ Not used by another result of the paper.

Proof

Formalized in Lean (see the linked declarations).

Evidence & scope

- **Program:** Bass's identity on K_3, exact (SymPy). `verify_jacobi_newton.py`

Theorem 4. Lean

($\mathrm{tr}(B^k)$ counts non-backtracking walks, in Lean 4)✓ p. 6

$$\mathrm{tr}(B^k) \;=\; \#\{\text{closed non-backtracking walks of length } k, \text{ rooted at a dart}\}.$$

sorry-free (`trace_hashimoto_pow`).

Lean · ours **show**

- `SimpleGraph.trace_hashimoto_pow`

✅ Checked when this page was built: NbWalkCount.lean compiles with Lean 4.35.0, no errors, no sorry. Axioms — trace_hashimoto_pow: propext, Classical.choice, Quot.sound. Reproduce: Lean 4.35.0, Mathlib f0b4c8150aa0; in that Lean project, `lake env lean NbWalkCount.lean` (SHA-256 of the compiled file: `234ce32e57f69492…`; a source page on this site may show it with comments edited, under its own hash).

Trust base: 🔒 kernel only. Lean: 1 result, standard axioms only

Proof: in this note · **Formalized:** Lean 4, by the author (public formalizations repository) · **Covers:** the whole statement · **Cite:** Theorem 4, p. 6, doi:10.5281/zenodo.20578470

↳ Not used by another result of the paper.

Proof

Formalized in Lean (see the linked declarations).

Evidence & scope

- **Program:** tr(B^k) against a brute-force count of rooted closed non-backtracking walks on K_3, k = 1..6: 0, 0, 6, 0, 0, 6. `verify_jacobi_newton.py`

Table 1: Every machine-checked ingredient. All are `sorry`-free; `#print axioms` reports only `[propext, Classical.choice, Quot.sound]`. Sources: `Ihara/TraceFormula.lean` and `MathlibPR/MatrixWalkCount.lean`.

2ggband!30white

Lean name	Statement
`Matrix.derivative_det`	$(\det M)' = \mathrm{tr}(\mathrm{adj}M \cdot M')$ (Jacobi
`one_sub_smul_mul_resolventSeries`	$(I - XM)\sum_k M^k X^k = I$
`trace_resolventSeries`	$\mathrm{tr}\sum_k M^k X^k = \sum_k \mathrm{tr}(M^k) X^k$

`Matrix.charpolyRev_logDeriv`	$(\mathrm{charpolyRev}\ M)' = -\ \mathrm{charpolyF}$ (Newton)
`pow_apply_eq_sum`	$(M^k)_{ij} = \sum_{\text{walks}} \prod(\text{weights})$
`relMatrix_pow_apply`	directed walk count, 0–1 case
`trace_pow_eq_sum_cyclicProd`	$\mathrm{tr}(M^{m+1}) = \sum_{\text{cyclic } w} \mathrm{cyclicProd}$
`cyclicProd_comp_finRotate_iterate`	rotation invariance (full group)
`ihara_Nk_explicit`	$\sum_k \mathrm{tr}(B^{k+1}) X^k$ from (A, D)
`trace_hashimoto_pow`	$\mathrm{tr}(B^k) = \#$ closed nb-walks

Colour: blue shade = the number's size within its column.

Table 2: Independent numerical cross-checks (`SymPy`), exact symbolic equality. The graph row is the triangle K_3: the trace of B^k matches a brute-force enumeration of rooted closed non-backtracking walks, $0, 0, 6, 0, 0, 6$ for $k = 1, \ldots, 6$ (the cycle and its reverse, three basepoints each).

2ggband!30white

Identity	Instance	Result
Jacobi $(\det M)' = \mathrm{tr}(\mathrm{adj}M\ M')$	3×3 matrix of polynomials	exact
Newton (log-derivative)	3×3 and 2×2, mod X^8	exact
$\mathrm{tr}(B^k) = \#\{\text{rooted nb-walks}\}$	K_3, $k = 1, \ldots, 6$	$0, 0, 6, 0, 0, 6$ both
Bass $(1-u^2)^{\lvert V\rvert} \det(I-uB) = \ldots$	K_3	exact

📊 Tables

Every table of the paper, in its order: click one to unfold it, with where the paper cites it, how its numbers were checked, and its data as CSV. The typeset table is one click away, on its page of the PDF.

Table 1. Every machine-checked ingredient. All are `sorry`-free; `#print axioms` reports only `[propext, Classical.choice, Quot.sound]`. Sources: `Ihara/TraceFormula.lean` and `MathlibPR/MatrixWalkCount.lean`.

whiteLean name	whiteStatement
`Matrix.derivative_det`	$(\det M)' = \mathrm{tr}(\mathrm{adj}M \cdot M')$ (Jacobi)
`one_sub_smul_mul_resolventSeries`	$(I - XM)\sum_k M^k X^k = I$

`trace_resolventSeries`	$\operatorname{tr}\sum_k M^k X^k = \sum_k \operatorname{tr}(M^k) X^k$
`Matrix.charpolyRev_logDeriv`	$(\operatorname{charpolyRev} M)' = -\operatorname{charpolyRev}$ (Newton)
`pow_apply_eq_sum`	$(M^k)_{ij} = \sum_{\text{walks}} \prod(\text{weights})$
`relMatrix_pow_apply`	directed walk count, 0–1 case
`trace_pow_eq_sum_cyclicProd`	$\operatorname{tr}(M^{m+1}) = \sum_{\text{cyclic } w} \operatorname{cyclicProd}$
`cyclicProd_comp_finRotate_iterate`	rotation invariance (full group)
`ihara_Nk_explicit`	$\sum_k \operatorname{tr}(B^{k+1}) X^k$ from (A, D)
`trace_hashimoto_pow`	$\operatorname{tr}(B^k) = \#$ closed nb-walks

Colour: blue shade = the number's size within its column.

p. 6 in the PDF | ⬇ CSV | not cited in the text

Table 2. Independent numerical cross-checks (SymPy), exact symbolic equality. The graph row is the triangle K_3: the trace of B^k matches a brute-force enumeration of rooted closed non-backtracking walks, $0, 0, 6, 0, 0, 6$ for $k = 1, \dots, 6$ (the cycle and its reverse, three basepoints each).

whiteIdentity	whiteInstance	whiteResult
Jacobi $(\det M)' = \operatorname{tr}(\operatorname{adj} M\, M')$	3×3 matrix of polynomials	exact
Newton (log-derivative)	3×3 and 2×2, mod X^8	exact
$\operatorname{tr}(B^k) = \#\{\text{rooted nb-walks}\}$	K_3, $k = 1, \dots, 6$	$0, 0, 6, 0, 0, 6$ both
Bass $(1-u^2)^{\lvert V\rvert} \det(I - uB) = \dots$	K_3	exact

p. 7 in the PDF | ⬇ CSV | cited in the opening, before §1

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