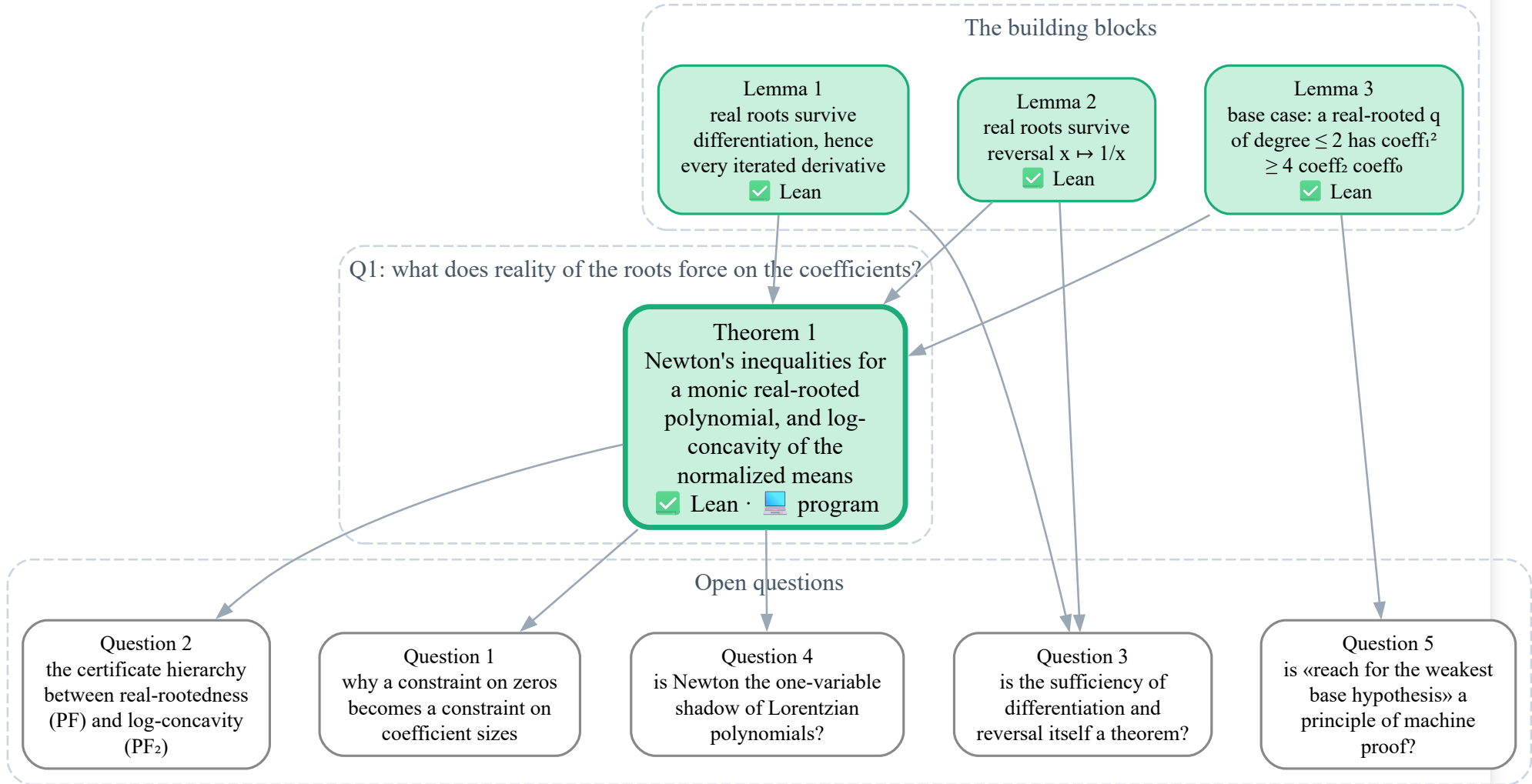
Map of *The Inward Bow of a Real-Rooted Polynomial: Newton's Inequalities, Machine-Checked in Lean 4*

Inequalities for polynomials · real-rooted polynomials and log-concavity

9 results | **4** entirely in Lean | **0** partly in Lean | **1** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

A machine-checked, `sorry`-free proof in Lean 4 over Mathlib of **Newton's inequalities** (1707): for a real-rooted real polynomial of degree n, $e_k^2 \cdot C(n,k-1) \cdot C(n,k+1) \geq e_{k-1} \cdot e_{k+1} \cdot C(n,k)^2$ for $1 \leq k \leq n-1$; equivalently the normalized means $p_k = e_k / C(n,k)$ are log-concave.

- **The theorem** (Theorem 1): the symmetric-function form assumes p monic; the coefficient form `newton_inequality_coeff` assumes nothing.
- **The route** is the classical one (Sylvester; Hardy–Littlewood–Pólya): differentiate, reverse, differentiate again, down to a real-rooted quadratic, whose discriminant is the inequality once positive factorials are cleared.
- **Four reusable blocks**: real-rootedness under differentiation, iterated differentiation and reversal, and the coefficient-form discriminant for degree ≤ 2, which also absorbs the degenerate case $a_{i-1} = 0$.
- **Boundary**: the mathematics is classical; the paper claims the formalization (to the author's knowledge the first in any proof assistant, June 2026). The equality cases and Maclaurin's chain are named, not proved.

🚀 **Start here:** **Theorem 1**: Newton's inequalities for a monic real-rooted polynomial, and log-concavity of the normalized means.

Main results:

- **Theorem 1**: Newton's inequalities for a monic real-rooted polynomial, and log-concavity of the normalized means

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Real-rootedness preservers** → Rolle's theorem for polynomials (real roots survive differentiation) and reversal $x \mapsto 1/x$ — Theorem 1
- **Symmetric functions** → Vieta's relations between coefficients and the elementary symmetric functions of the roots (Mathlib's Multiset.esymm) — Theorem 1
- **Formal verification** → Lean 4 + Mathlib: falling factorials of iterated derivatives, exact degrees in characteristic zero — Theorem 1

What the results serve

- **Log-concavity in combinatorics** ← real-rooted ⇒ log-concave coefficients, e.g. the matching numbers through Heilmann–Lieb (named in §4, not composed here)

— Theorem 1

- **Maclaurin's inequalities and the equality cases** ← the next targets the paper names; not proved