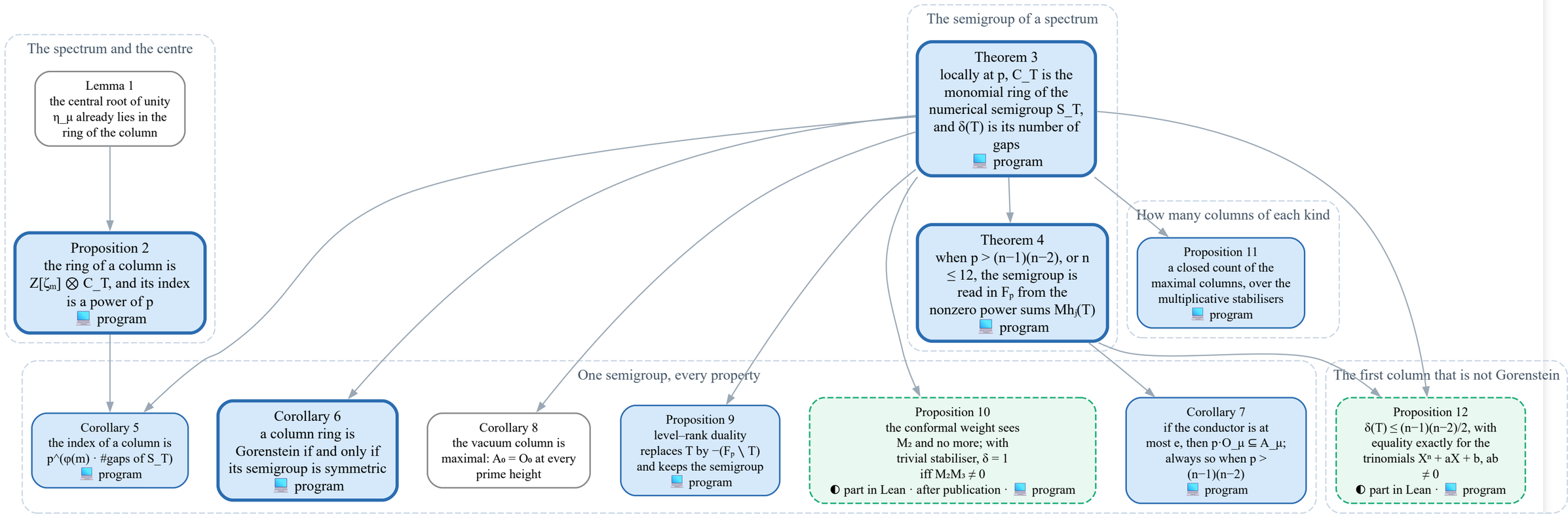
Map of *The order generated by a column of the $SU(n)_k$ S-matrix is locally a numerical semigroup ring (Part I)*

Algebraic number theory · orders generated by character values

12 results | **0** entirely in Lean | **2** partly in Lean | **10** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

A column μ of the modular S-matrix of $SU(n)_k$, normalised by its first entry, generates a ring A_μ of character values. Its field was known (Gannon–Schopieray); the paper determines the **ring** when the height $p = n + k$ is prime.

- **The decomposition** (Lemma 1, Proposition 2): $A_\mu = \mathbb{Z}[\zeta_m] \otimes C_T$, a maximal cyclotomic factor fixed by the charge under the centre, and an order C_T that depends only on the reduced spectrum $T \subset \mathbb{F}_p$; the index is a power of p.
- **The exact semigroup** (Theorem 3, Corollary 5): locally at p, C_T is a monomial ring whose valuations form a numerical semigroup S_T, written by Gauss sums and Stickelberger's theorem; the index is $p^{\varphi(m)\delta}$, δ the number of gaps.
- **Read in $\mathbb{F}_p$** (Theorem 4): when $p > (n-1)(n-2)$, and for every p when $n \le 12$, S_T is generated by e and the j with $M_{hj}(T) \neq 0$.
- **One semigroup, every property** (Corollaries 6–8, Propositions 9–10): Gorenstein ⇔ S_T symmetric (Kunz), exponent p when the conductor is small, the vacuum is maximal, level–rank keeps S_T, the conformal weight sees M_2.
- **Counts and extremes** (Propositions 11–12, §6): a closed count of the maximal columns, the largest defect $(n-1)(n-2)/2$ attained exactly by the trinomials $X^n + aX + b$, and the first non-Gorenstein column at prime height, $SU(5)_{12}$.

🚀 **Start here: Proposition 2**: the ring of a column $Z[\zeta_m] \otimes C_T$, and its index is a power of p.

Main results:

- **Proposition 2**: the ring of a column is $Z[\zeta_m] \otimes C_T$, and its index is a power of p
- **Theorem 3**: locally at p, C_T is the monomial ring of the numerical semigroup S_T, and δ(T) is its number of gaps
- **Theorem 4**: when $p > (n-1)(n-2)$, or $n \le 12$, the semigroup is read in F_p from the nonzero power sums $Mh_j(T)$
- **Corollary 6**: a column ring is Gorenstein if and only if its semigroup is symmetric

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Gauss sums** → Stickelberger's valuations of Gauss sums and Galois eigenprojectors in a tamely ramified extension — Theorem 3
- **Numerical semigroups** → Selmer's formula for the genus, Schur's bound on the conductor (via Brauer), Kunz's Gorenstein criterion, Apéry sets — Theorem 3,

Theorem 4, Corollary 6, Proposition 12

- **Symmetric functions** → Newton's identities, complete symmetric functions, complementary partitions in a box (Macdonald) — Lemma 1, Theorem 4, Proposition 9
- **Polynomials over finite fields** → Bank, Bary-Soroker and Rosenzweig: factorisation types in short intervals, for the existence of split trinomials — Proposition 12

What the results serve

- **Modular data and fusion rings** ← the arithmetic of one column of the SU(n)_k S-matrix: index, Gorenstein property, level-rank duality, conformal weight — Corollary 5, Corollary 6, Proposition 9, Proposition 10
- **Orders generated by character values** ← the order (not only the field) at an element of finite order of SU(n), in the line of Bächle and Sambale — Proposition 2, Corollary 5
- **The gluing of the columns (Part II)** ← the defect δ(T) = number of gaps of S_T is the per-column term of the discriminant of the fusion ring — Theorem 3, Proposition 12