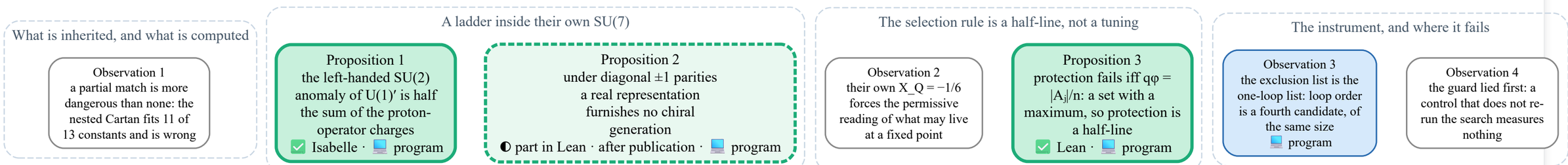
Map of *Proton Decay in SU(7) Grand Gauge-Higgs Unification: An Obstruction, Its Minimal Escapes, and the One Row of Their Table 1 That Can Pay for Them (Part VI)*

High-energy physics · grand gauge-Higgs unification

7 results | ✅ **1** entirely in Lean | ◐ **1** partly in Lean | ✅ **1** in Isabelle | 💻 **4** checked by a program | written proof only | (a result can carry several tags)

Dependencies in the written proof: an arrow A → B means that this proof of B uses A. Every result rests on its written proof; the tags record what was checked besides, and each statement's "Evidence & scope" block says how far. The colour of a box is its strongest tag, in the order above; the tags in the box give the rest. Boxes with a thick border are the main results; click a box to read the statement.



What the paper contributes

Komori and Maru's six-dimensional SU(7) grand gauge-Higgs model leaves proton decay to future work. Working inside their construction and with their own equations, the paper finds an obstruction, classifies the minimal escapes and selects one row of their Table 1 as the benchmark to compute next; it does not deliver that model.

- **An obstruction at one generation.** Three anomaly channels force $X_Q = -1/6$, where $U(1)' = T_{3L} + Y - (B - L)$ identically and the four dimension-6 $|\Delta B| = 1$ operators are neutral; two further channels require a right-handed neutrino of charge -1, which only a **28** supplies (rows (2) and (3)).
- **The escapes, inside their own group.** A ladder extra(L) = $(1 - k)/2$ and the identity $[SU(2)_L]^2 U(1)' = \tfrac{1}{2} \Sigma_j A_j$ (Proposition 1): fourteen anomaly-free three-generation assignments, two of which fit their tensors. The **48** cannot host a generation, because under diagonal ± 1 parities a real representation is vector-like (Proposition 2, a specialisation of Gogoladze–Mimura–Nandi).
- **The bill, in eighths.** $V''(0) = -\pi^2 \zeta(3) D$ with D rational per multiplet ($\mathbf{84}^{(+,+)} \mapsto 5/4$, $\mathbf{48}^{(+,+)} \mapsto 0$): only case (2) satisfies both necessary conditions, at one-loop curvature.
- **A selection rule in closed form.** Protection fails iff $q_\varphi = |A_j|/n$ (Proposition 3); $q_\varphi = 3/2$ protects for every half-integer brane-quark charge.
- **The instrument, audited.** Their α_{min} column is not reproduced (an open question, not a defect claim); the verdict holds on a wedge of multiplicative repairs, and two-loop safety is left open.

🚀 **Start here:** **Proposition 1**: the left-handed SU(2) anomaly of $U(1)'$ is half the sum of the proton-operator charges.

Main results:

- **Proposition 1**: the left-handed SU(2) anomaly of $U(1)'$ is half the sum of the proton-operator charges
- **Proposition 2**: under diagonal ± 1 parities a real representation furnishes no chiral generation
- **Proposition 3**: protection fails iff $q\varphi = |A_j|/n$: a set with a maximum, so protection is a half-line

🧭 Fields: where the paper lives and what it connects

Tools taken from other branches

- **Anomaly cancellation** → the six $U(1)'$ anomaly channels on the published content; the gravitational and cubic conditions as a Diophantine system (Costa–Dobrescu–

Fox) — Proposition 1
- **Representation theory of SU(7)** → weights of tensor powers and of the adjoint, orbifold parities, chiral zero modes — Proposition 1, Proposition 2
- **Discrete gauge symmetry** → the residual subgroup left by a charged condensate (Krauss–Wilczek, Ibáñez–Ross) — Proposition 3

What the results serve

- **Proton stability in orbifold GUTs** ← the obstruction at one generation, the family-dependent escapes, and the selection rule for the breaking scalar — Proposition 1, Proposition 3
- **Model building in gauge-Higgs unification** ← case (2) of Komori–Maru Table 1 as the benchmark to compute next (Part VII of the series takes it up)